

Counting the Cycles of Light using a Self-Referenced Optical Microresonator

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Phase coherently linking optical to radio frequencies with femtosecond mode-locked laser frequency combs enabled counting the cycles of light and is the basis of optical clocks, absolute frequency synthesis, tests of fundamental physics, and improved spectroscopy. Using an optical microresonator frequency comb to establish a coherent link between optical and microwave frequencies will extend optical frequency synthesis and measurements to areas requiring compact form factor, on chip integration and comb line spacing in the microwave regime, including coherent telecommunications, astrophysical spectrometer calibration or microwave photonics. Here we demonstrate a microwave to optical link with a microresonator. Using a temporal dissipative single soliton state in an ultra-high Q crystalline microresonator that is broadened in highly nonlinear fiber an optical frequency comb is generated that is self-referenced, allowing to phase coherently link a 190 THz optical carrier directly to a 14 GHz microwave frequency. Our work demonstrates precision optical frequency measurements can be realized with compact high Q microresonators.

The development of the optical frequency comb (OFC) based on femtosecond pulsed mode-locked lasers [1–3] in conjunction with nonlinear spectral broadening constituted a dramatic simplification over large scale harmonic frequency chains [4]. In the frequency domain OFCs give an equidistant optical lines, where the frequency of each component obeys $f_n = n \cdot f_{rep} + f_0$. The spacing between the lines is determined by the pulse repetition rate of the laser f_{rep} (n being an integer). Knowledge of the comb's overall offset frequency f_0 (also referred to as the carrier envelope offset frequency) along with f_{rep} and n allows linking the optical frequency f_n to the electronically countable frequency f_{rep} and f_0 in the radio frequency or microwave domain. In this way the measurement of f_{rep} and f_0 corresponds to counting the cycles of light. Self-referencing of the comb, i.e. a self-contained measurement of f_0 and f_{rep} , has been achieved using nonlinear interferometers, whereby the combs bandwidth needs to be broadened to encompass typically two-thirds of or a full octave [5–8]. This measurement is a key prerequisite for many applications of OFCs. For example, referencing one of the comb lines of an OFC to an atomic frequency standard [9] allows the comb to function as a 'gearbox' realizing the next generation of atomic clocks based on optical transitions. Self-referenced frequency combs can also be used for optical frequency synthesis, and have enabled the most accurate frequency measurements [10, 11]. Frequency combs with large mode spacings (≥ 10 GHz), which are challenging to obtain via mode-locked lasers [12], can be used in a growing number of applications, including astronomical spectrometer calibration [13], dual comb coherent Raman imaging [14], high speed optical sampling or coherent telecommunications [15]. In each of these applications, having a line spacing $\gtrsim 10$ GHz is either beneficial or even required.

Different from mode-locked lasers, microresonator frequency combs (MFC) [16, 17] are generated using para-

metric frequency conversion [18, 19], of a continuous wave (CW) laser. This approach exhibits several attractive features that have the potential to extend further the use of OFCs to new areas in precision optical measurement, spectroscopy, astronomy, telecommunications and industrial applications. Fundamentally different from mode-locked laser systems, MFCs offer: high repetition rates (> 10 GHz), compact form factor, broadband parametric gain that can be generated in wavelength regimes ranging from the visible [20] to the mid-infrared [21, 22], CMOS compatible microresonator platforms, and high power per comb line. A unique characteristic of MFCs is the pump laser constitutes one of the frequency comb components and there is no active gain laser medium in the system. These properties have encouraged in recent years the intense investigation of microresonator based frequency combs with the ultimate goal of realizing a self-referenced system. Progress in recent years includes new microresonator platforms in crystalline materials [23], fused silica microtoroids [16], and photonic chips (based on Silicon Nitride [24, 25], Aluminum Nitride [26], Hydrex[®] [27, 28] and diamond [29]). In addition, MFCs without self-referencing have been used for coherent telecommunications [15], compact atomic clocks [30], stabilized oscillators [31], and optical pulse generation [32–34]. The dynamics of microresonator frequency combs have been investigated, and regimes with low noise frequency comb operation have been identified based on intrinsic low phase noise regimes, or via tuning mechanisms such as $\delta - \Delta$ matching [35], parametric seeding [36], injection locking [36, 37] or via the observation of phase-locking [33]. Moreover recently, low noise frequency combs have been generated via temporal dissipative soliton formation [34, 38–40] and numerical tools to simulate comb dynamics emerged based on the Lugiato-Lefever equation [41–43] and the coupled-modes equation [34, 44].

However, despite the rapid progress in understanding, simulations, applications and platforms, an outstanding milestone has not been reached: a self-referenced microresonator system, capable of phase coherently linking

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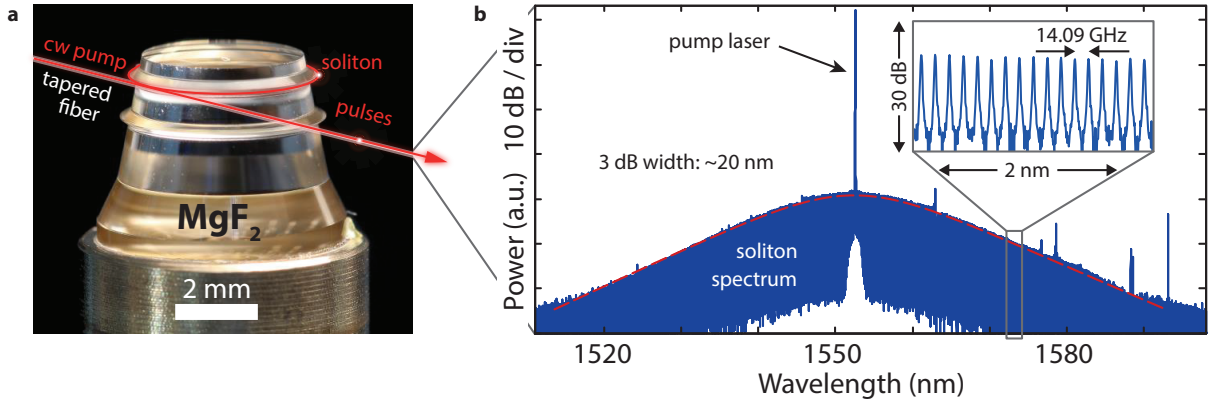


Figure 1. Crystalline MgF_2 microresonator and temporal dissipative soliton generation: *a*. Optical image of the employed ultra high Q crystalline whispering gallery optical microresonators on a magnesium fluoride pillar with a diameter of several millimeters. The ultra high Q whispering gallery optical modes are confined in the fabricated protrusions that extend around the circumference. The top resonator was used in the experiments and the mode of interest has a free spectral range of 14.0939 GHz and a loaded $Q \approx 10^9$. *b*. The hyperbolic-secant shaped spectrum (fit: red dotted line) of the single temporal soliton produced inside the resonator by the continuous wave pump laser. The inset shows the ability to resolve the microresonator comb lines on a grating based spectrometer.

the microwave and the optical frequency domain. So far, knowledge of the absolute frequency of all comb lines of a MFC has only been achieved using an auxiliary self-referenced fiber laser frequency comb as a reference [45], as well as via locking of two comb teeth to a Rubidium transition [30]. Self-referencing however, has never been achieved. One reason for this is the high repetition rate of the microresonators leading to correspondingly low pulse peak intensities. This makes external spectral broadening using nonlinear fiber, which is widely employed in mode-locked lasers, difficult to apply. While octave spanning combs [46, 47] have been attained directly from MFCs, they have not been suitable for self-referencing techniques due to excess noise associated with subcomb formation [35].

Here, we demonstrate a coherent microwave to optical link using temporal dissipative soliton formation in a crystalline microresonator in conjunction with external spectral broadening, measuring simultaneously both f_{rep} and f_0 that are necessary for linking the optical to the radio frequency domain. Our approach uses the newly discovered class of temporal dissipative cavity solitons in microresonators [34, 39, 48, 49] and marks the first time that the carrier envelope offset frequency of such a soliton has been measured. Our results demonstrate that MFCs generated by soliton formation are suitable for absolute frequency measurements. In particular the pulse to pulse timing jitter is sufficiently low to enable precise measurement of the carrier envelope offset frequency via self-referencing. Although not demonstrated here, it has already been shown that microresonator comb parameters f_{rep} and f_0 can be stabilized by controlling the pump laser frequency and by actuating the resonator free spectral range by heating or mechanical stress [45, 50].

Optical microresonators support different azimuthal

optical whispering gallery modes (WGM). The free spectral range (FSR) between modes of a particular mode family is determined by material and geometric dispersion. It has been shown that WGM can have quality factors exceeding 10^{10} [51, 52]. When a CW pump laser is coupled to a WGM the resonator's Kerr nonlinearity can lead to the efficient nonlinear frequency conversion. The resonator used in this work is a crystalline ultra high Q resonator made by polishing crystalline MgF_2 [53–55] (cf. figure 1), and can support WGMs [51] confined in one of its protrusions that extend around the circumference of the resonator. The mode used has a quality factor of $\sim 10^9$ and a free spectral range of 14.0939 GHz. Light from a continuous wave fiber laser at 1553 nm with ~ 150 mW can be coupled into and out of the resonator via evanescent coupling using a tapered optical fiber [56]. To form the solitons in the cavity, the pump laser's frequency is scanned over the resonance and stopped when the appropriate conditions are met [34]. The duration of the pulse inside the resonator depends mainly on the coupling and detuning of the pump laser to the resonator mode, and can be estimated from the bandwidth of the spectrum shown in figure 1, to be ~ 130 fs. The optical spectrum generated by the soliton is not yet sufficiently broad for self-referencing; however, the spectrum can be broadened via supercontinuum generation [57].

The experimental setup after the resonator is shown in figure 2. A portion of the pulse train produced by the resonator is sent to an optical spectrum analyzer (OSA). The rest of the pulse train then has the CW background consisting of the residual pump laser [34] minimized by fiber optic filters. Then another small amount is sent to a photodetector and a electronic spectrum analyzers (ESA) to detect f_{rep} . The pulse is then preamplified with an erbium doped fiber amplifier (EDFA). The main type

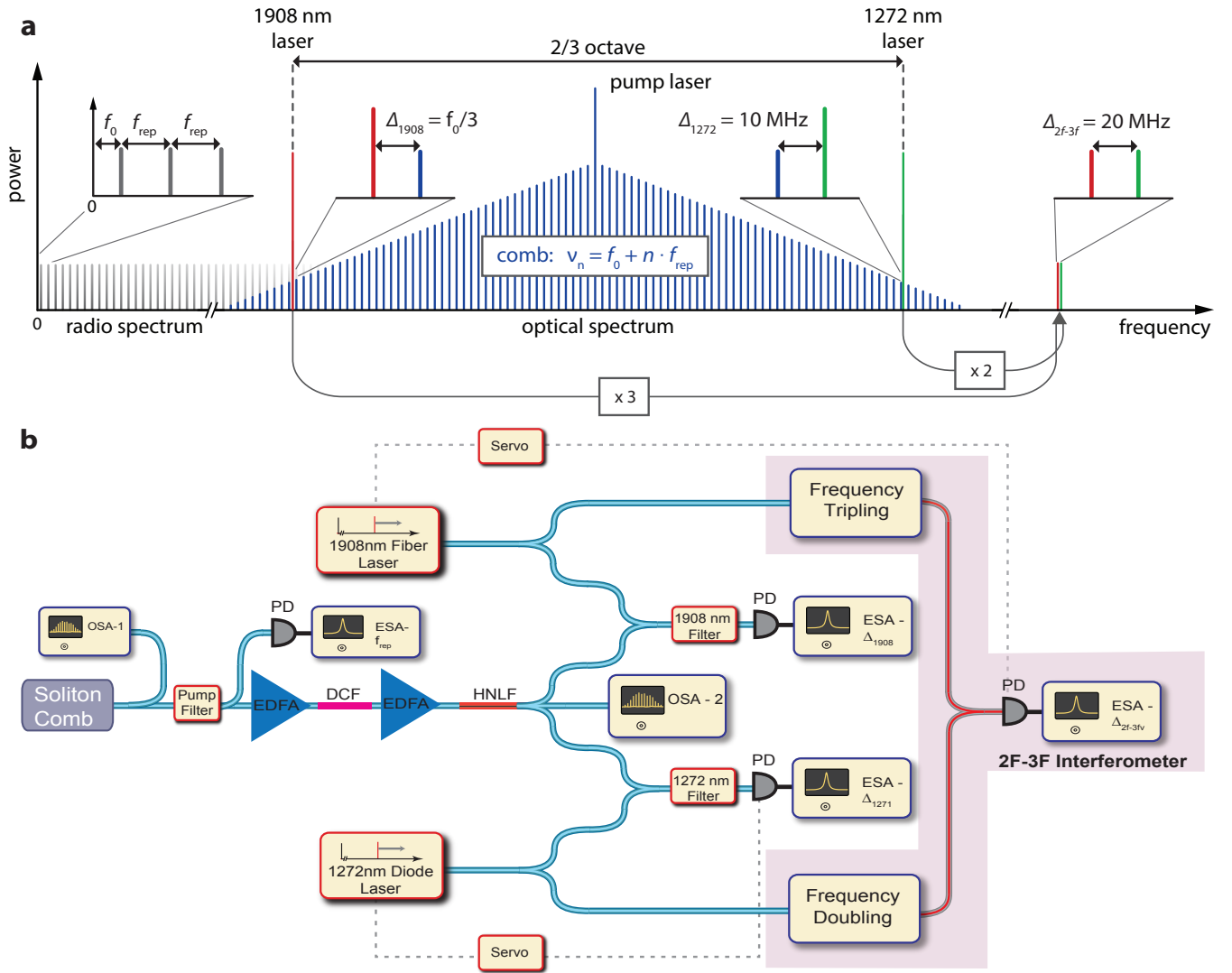


Figure 2. Experimental setup and $2f - 3f$ microresonator self-referencing scheme: *a*. The frequency domain picture showing the relevant frequency components used to self-reference the comb and to determine the carrier envelope offset frequency (f_0). *b*. The simplified experimental setup used for self-referencing. A portion of the solitons that are outcoupled from the resonator are sent to an OSA to measure the spectrum, then residual pump light is filtered out using fiber optic filters before a portion is picked off and sent to a photodetector (PD) to measure the repetition rate on an electronic spectrum analyzer (ESA). The pulse is first preamplified in an erbium doped fiber amplifier (EDFA) and then prechirped to temporally broaden it with a dispersion compensating fiber (DCF) before being amplified by a high power EDFA. The pulse is subsequently recompressed and coupled into a highly nonlinear fiber (HNLf) where the coherent supercontinuum is generated. A fraction of the spectrum is mixed with light from the 1908 nm Thulium fiber laser and sent through a 1908 nm bandpass filter to a PD and ESA to measure Δ_{1908} . The same is done with the 1272 nm external cavity diode laser to measure Δ_{1272} and a servo loop is used to phase lock the laser to the optical frequency comb and fix Δ_{1272} where a signal from an atomic clock is used as a reference. To create the $2f - 3f$ interferometer light from the 1272 nm laser is frequency doubled in a periodically poled Lithium Niobate crystal (PPLN) to produce light at 636 nm. Light from the 1908 nm laser is frequency doubled to 954 nm in a PPLN crystal, and subsequently combined with 1908 nm and sent through a PPLN crystal phase matched for sum frequency generation creating light at 636 nm. The generated visible light is optically heterodyned on a PD with the frequency doubled light from the 1272 nm laser, permitting to measure Δ_{2f-3f} on an ESA. This offset frequency is fixed by phase locking the 1908 nm laser via the $2f - 3f$ interferometer. With this scheme the carrier envelope frequency is measured by recording Δ_{1908} .

of fiber in the experiment is SMF28, which has anomalous group velocity dispersion in the wave length range of the soliton. Before being sent into a high EDFA the soliton pulse is prechirped using dispersion compensating

fiber (DCF), which has normal group velocity dispersion, to minimize nonlinear effects in the EDFA. In this way the average power of the pulse train is increased to 2 W. After the EDFA, the pulse is recompressed using SMF28

fiber via the cut back method to a duration of ~ 300 fs with an energy of ~ 150 pJ. This pulse is subsequently sent through approximately 2 m of highly nonlinear fiber (HNLf)(Menlo Systems), where supercontinuum generation occurs. The resulting spectrum can be seen in figure 3. The blue trace shows the soliton spectrum after the optical microresonator, and the red after the HNLf fiber. The latter spectrum exceeds two-thirds of an octave, which is sufficient for self-referencing via a $2f - 3f$ interferometer [5–8]. Importantly, the broadened spectrum is coherent. This is verified by using a heterodyne beat with additional external lasers at the two ends of the comb, as detailed below. Figure 3b shows a zoom into the spectrum taken after the HNLf fiber where the individual comb lines are clearly visible, even with the limited resolution of the OSA (0.02 nm).

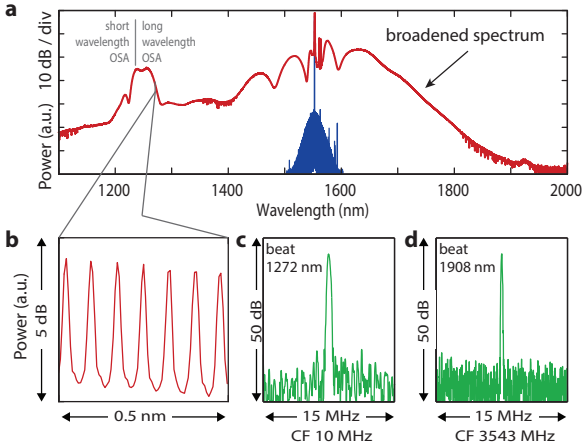


Figure 3. Optical spectrum of the microresonator before and after external broadening: *a*. The blue trace shows the optical spectrum generated in the crystalline optical microresonator by the temporal dissipative soliton state. The large central spike originates from residual light from the pump laser. The spectrum after the supercontinuum generation is denoted in red. It is composed of data take from two different OSAs as result of the limited bandwidth of the individual instruments. *b*. Zoom into the broadened spectrum revealing the widely spaced comb lines. *c*. Heterodyne beatnote of a laser at 1272 nm with the broadened comb demonstrating a signal to noise ratio exceeding 40 dB in the resolution bandwidth (RBW) of 300 kHz. *d*. shows heterodyne beatnote at the long wavelength end of the comb at 1900 nm (RBW 100 kHz).

Self-referencing is achieved by measuring f_{rep} and f_0 of the generated frequency comb. By picking off a small portion of the light after it leaves the resonator and sending it to a photodetector f_{rep} can be directly measured (cf. figure 2). The broadened spectrum is sufficiently wide and allows employing a $2f - 3f$ interferometer [5] to determine f_0 . Traditionally this is implemented by frequency tripling a component of the low frequency part of the spectrum $f_L = n \cdot f_{rep} + f_0$ using a combination of second harmonic and sum frequency generation to give $3f_L = 3n \cdot f_{rep} + 3f_0$, where n is an integer. In addition, a component of the higher frequency part of the spectrum

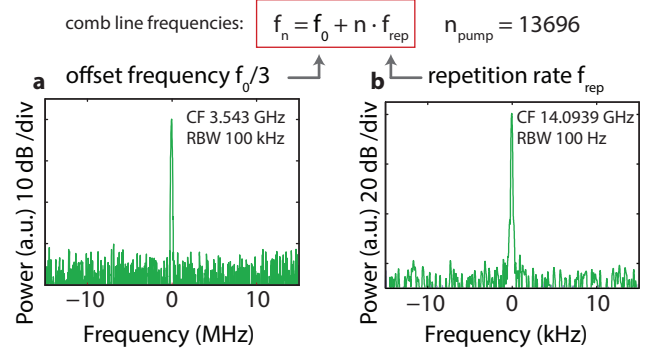


Figure 4. Repetition rate and carrier envelope frequency signals of the self-referenced microresonator comb: *a*. The offset frequency (f_0) of the optical microresonator frequency comb divided by three as described in eqn. 7. The measured optical heterodyne beat frequency has a center frequency of 3.543 GHz and exhibits a signal to noise that exceeds 30 dB in a 100 kHz RBW. *b*. The repetition rate f_{rep} of the soliton in the optical microresonator with a center frequency (CF) of 14.0939 GHz and a signal to noise ratio (SNR) > 60 dB measured in a resolution bandwidth of 100 Hz. The large SNRs are sufficient for accurate phase tracking of the two microwave signals.

$f_H = m \cdot f_{rep} + f_0$ is frequency doubled using second harmonic generation to give $2f_H = 2m \cdot f_{rep} + 2f_0$ where m is an integer. Mixing the doubled and the tripled light and detection on a photodetector gives the offset frequency $3f_L - 2f_H = f_0$, granted $3n = 2m$, i.e. necessitating a spectrum that covers two-thirds of an octave. Here a scheme involving two transfer lasers is implemented, which has the advantage of allowing independent verification of the coherence of the supercontinuum generation at the two ends of the spectrum. The coherence of the generated broadband OFC is verified by optically heterodyning the two reference lasers at ~ 1272 nm (external cavity diode laser) and ~ 1908 nm (Thulium fiber laser) with the supercontinuum and detecting the optical heterodyne beat signal on a photodetector (see figure 3). The beatnote frequencies can be written in terms of the frequency comb parameters and an offset as

$$f_{1272} = n \cdot f_{rep} + f_0 + \Delta_{1272} \quad (1)$$

and

$$f_{1908} = m \cdot f_{rep} + f_0 - \Delta_{1908} \quad (2)$$

where n and m are integers and Δ_{1272} and Δ_{1908} are the frequency offsets (> 0) of the transfer lasers from the generated OFC (see figure 3). The offset frequency Δ_{1272} and Δ_{1908} are chosen to be both positive by convention. The $2f - 3f$ interferometry is constructed with the reference lasers where one arm of the interferometer serves for second harmonic generation of the light at 1272 nm to give light at 636 nm written as (see figure 2)

$$2f_{1272} = 2n \cdot f_{rep} + 2f_0 + 2\Delta_{1272}. \quad (3)$$

The other interferometer arm serves for frequency tripling of the light at 1908 nm via second harmonic generation to create light at 954 nm followed by sum frequency generation of the 954 nm and 1908 nm light to give light at 636 nm, and the frequency can be written as:

$$3f_{1908} = 3m \cdot f_{rep} + 3f_0 - 3\Delta_{1908}. \quad (4)$$

The doubled and tripled light are mixed and detected on a photodetector, giving an optical heterodyne signal at the difference frequency:

$$\Delta_{2f-3f} = 2f_{1272} - 3f_{1908}. \quad (5)$$

The offset frequency f_0 is related to the beats of the transfer lasers with the generated frequency comb:

$$\Delta_{2f-3f} = (2n - 3m) f_{rep} - f_0 - 3\Delta_{1908} + 2\Delta_{1272}. \quad (6)$$

The transfer laser at 1272 nm is phase locked to a frequency comb component with $\Delta_{1272} = 10$ MHz offset frequency. The frequency tripled 1908 nm transfer laser is phase locked via the $2f - 3f$ interferometer signal to 20 MHz below the the frequency doubled 1272 nm transfer laser at $\Delta_{2f-3f} = 20$ MHz. Both phase locks are referenced to a commercial atomic clock. For $2n - 3m = 0$ (which can readily be achieved by locking the 1272 nm transfer laser to the appropriate comb line) the beat Δ_{1908} between the 1908 nm transfer laser and the OFC corresponds to:

$$\Delta_{1908} = \frac{f_0}{3} \quad (7)$$

With this measurement technique an offset frequency of $\frac{f_0}{3} = 3.543$ GHz (see figure 4). The signal to noise ratio (SNR) of f_0 of >30 dB in 100 kHz resolution bandwidth (RBW), as well as a SNR of >60 dB in 100 Hz RBW of f_{rep} , is sufficient for accurate, i.e. real time counting of the cycles of the two radio frequency beats (and making the use of e.g. tracking oscillators unnecessary). This, along with the knowledge of the comb teeth number, provides the ability to directly count the cycles of the pump laser as well as the other comb teeth.

It should be noted, that the determination of the comb line index with a mode-locked laser frequency comb with repetition rates < 1 GHz, can be challenging. Here the much denser comb spectrum does normally not permit resolving the comb line with a grating based OSA, and moreover, the sensitivity to drifts in repetition rate is significantly higher, necessitating to fully phase stabilize the comb in order to determine the comb line index. In contrast, the comb line index can be obtained in the present work with a low resolution wavemeter without requiring locking of either f_{rep} or f_0 . In this way the comb line number of the pump laser was determined to be $n_{pump} = 13696$. Self referencing along with knowledge of the comb line numbers, implies that the absolute laser

frequency of the pump laser (f_{pump} , as well as any other comb teeth) can be directly determined via the repetition rate beatnote (f_{rep}) and the recorded beatnote (Δ_{1908}), since the absolute laser frequency is related to the two quantities via

$$f_{pump} = 3\Delta_{1908} + n_{pump} \cdot f_{rep}, \quad (8)$$

establishing the ability to count the cycles of light via the crystalline microresonator.

Our results constitute the first phase coherent link from the microwave to optical domain using a microresonator, by demonstrating measurement of the carrier envelope frequency of a temporal dissipative soliton in a microresonator. These results demonstrate microresonator based frequency combs can provide accurate and precise absolute optical frequency standards for a wide range of applications in optical frequency metrology, optical atomic clocks, optical frequency synthesis or low noise microwave generation by frequency division. In terms of soliton dynamics, this demonstration constitutes the first measurement of the carrier envelope offset frequency of a temporal dissipative Kerr cavity soliton. This opens a new route to studying nonlinear dynamics of solitons. A further important step in the future will be to phase lock both f_{rep} and f_0 to an external radio frequency reference, and thereby achieve a phase stabilized self-referenced microresonator frequency comb. This can be achieved by controlling both f_{rep} and f_0 independently via changing the pump frequency detuning along with either the pump power [45] or by applying a stress to the resonator with a piezoelectric crystal [50]. The crystalline microresonator based microwave to optical link can be made more compact with almost all optical components being fiber optic based, and aside from the optical microresonator the non-fiber based components (filters and sum frequency generation stages) can be replaced with fiber based components in the future. Finally, the external broadening stage itself, required in the present case, can in suitably dispersion engineered optical microresonators be avoided, when making use of soliton induced higher order spectral broadening [42, 47, 58]. Eventually, this provides a path to counting the cycles of light using chip-scale microresonators.

FUNDING INFORMATION

This work was supported by a Marie Curie IIF (J. D. J.), the Swiss National Science Foundation (T. H.), the European Space Agency (V. B.), a Marie Curie IEF (C. L.), the Eurostars program, and the Defense Advanced Research Program Agency (DARPA) PULSE program, grant number W31P4Q-13-1-0016.

ACKNOWLEDGMENTS

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